Empirical modelling of flow through a wind farm for comparison with wake model

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Abstract The goal of this project is to create a model that is able to predict the power output of the wind farm based on real world data. Initially, a physical approach was used by studying and implementing wake models available in literature. Then, its results and reasons why it failed to produce an accurate prediction are briefly studied. Afterwards, the creation of a second model based on empirical data is described. The new results are subsequently uncovered and described in order to understand its pros and cons and suggest where to act in order to possibly obtain a more accurate model. As a next step, the creation of a set of equations able to predict the power output of a single turbine using only wind speed and direction as inputs is attempted. Finally, a third model predicting directly the power output based on the wind speed and direction is formulated. The outcome of this last model is finally shown and compared with the results from the previous one.

1. Introduction

Wind energy is continuously increasing its share of installed power and energy produced worldwide. However, because of the stochastic nature of the wind resource, it results difficult to predict the power output of a wind farm. Creating a model that predicts the output of the wind farm would help in this aspect and consequently give additional stability to the grid. The physical and the empirical model that will be explained later aimed at finding a series of normalised velocity deficits $\Delta\overline{U}$ that would allow to find the wind speed at the location of each turbine according to equation (1).

$$u = U_\infty \cdot (1 - \Delta\overline{U}) \tag{1}$$

On the other hand the third model here formulated predicts directly the power output of the whole wind farm.

1.1. Data

The data covered the 12 months of 2016 in 10 minute increments. Out of these 12 months, 9 were used for the training of the model, while the remaining 3 for its validation. The wind direction was described starting from North ($0°$) and going clockwise (East is $90°$).

2. Background

An extremely important aspect that influences the power output and the layout of a wind farm is the wake effect that happens behind a rotating turbine. Several models describing it exist, but not all of them can be implemented in the models created in this project.

2.1. Wake Models

Jensen Model The first model created with the goal of describing the wake behind a rotor is the Jensen model [1], formulated in 1983. This model was then improved by I. Katic [2]. The Jensen model is used to describe the speed deficit depending only on the downstream distance x. For this reason, the Jensen model creates a top-hat speed profile and is also defined as a one-dimensional model. In this model, the wake expands linearly according to equation (2).

$$D_w = D \cdot (1 + 2ks) \tag{2}$$

with $s = x/D$ and k defining the wake decay coefficient, which is usually equal to 0.04 and 0.075 for offshore and onshore sites respectively [3].

The wind speed u at a certain distance x (or s if normalised) from the rotor is estimated with equation (3).

$$u = U_\infty \cdot \left[1 - \frac{1 - \sqrt{1 - C_T}}{(1 + 2 \cdot k \cdot s)^2}\right] \tag{3}$$

Ainslie Model The next attempt to describe the wake created by a wind turbine was made by J. F. Ainslie in 1988 [4]. This model is based on time averaged Navier-Stokes equations for an incompressible flow. The main objective of this model is not to provide an easily-applicable model (like the Jensen model), but to thoroughly describe the behaviour of the wake behind a turbine [5]. For this reason, it does not have an immediate equation to use in order to estimate the deficit. The speed profile obtained from this model does not vary only with x but also with the distance from the centreline of the wake r.

Larsen Model The first version of the Larsen model was published in 1988 [6], an improved one was presented in 1996 [7] and the present version was revealed in 2009 [8]. Like the Ainslie model, the velocity profile described by the Larsen model vary along the downstream direction and the radius of the wake. The radius of the wake is defined in (4):

$$R_w(x) = \left(\frac{105c_1^2}{2\pi}\right)^{1/5} \cdot (C_T \cdot A(x+x_0))^{1/3} \tag{4}$$

For the formulation of x_0, c_1 and the speed deficit $\Delta\overline{U}$ the reader is suggested to go through the original article [8].

Frandsen Model The Frandsen model was presented in 2006 [9] and is used for large offshore wind farms. It produces a top-hat velocity profile. In its most recent version of the model, the diameter of the wake is defined as in (5) [10].

$$D_w(x) = D \cdot max[\beta, \alpha \cdot s]^{1/2} \tag{5}$$

with $\alpha = 0.7$ and $\beta = \frac{1+\sqrt{1-C_T}}{2\cdot\sqrt{1-C_T}}$. The speed within the wake is estimated with equation (6), where $A_{w,0} = A \cdot \beta$.

$$u = \frac{U_\infty}{2} \cdot \left(1 \pm \sqrt{1 - 2 \cdot \frac{A}{A_w} \cdot C_T}\right) \tag{6}$$

Where $+$ is used when $C_T \leq 0.75$ and $-$ when $C_T > 0.75$.

Gaussian Wake Model A more recent model is the Gaussian Wake Model defined in 2014 [11]. This model requires the use of an expansion rate k^* which is site-dependant and varies with the surface roughness and the turbulence intensity. The normalised velocity deficit is calculated using equation (7):

$$\frac{\Delta\overline{U}}{U_\infty} = C(x) \cdot exp\left(\frac{-r^2}{2\cdot\sigma^2}\right) \tag{7}$$

with $C(x) = 1 - \sqrt{1 - \frac{C_T}{8\cdot(\sigma/D)^2}}$

2.2. Wake Combination Models

When using a wake model for a whole wind farm, it is necessary to find a way to merge the several wakes created by the turbines. There are four methods that have been tested for this purpose.

Linear Superposition This method finds the overall deficit by making the sum of the deficits of all the wakes.

$$\Delta\overline{U}_{n+1} = \sum_{j=1}^{n} \left(\Delta\overline{U}_j|_{x(n+1)}\right) \tag{8}$$

Quadratic superposition This model is widely used and was proposed by Katic when improving the Jensen model [2].

$$\Delta\overline{U}_{n+1} = \sqrt{\sum_{j=1}^{n} \left(\Delta\overline{U}_j|_{x(n+1)}\right)^2} \tag{9}$$

Superposition Within the Dynamic Wake Meandering Approach This approach assumes that the wakes intercepting the rotor are independent of each other. Consequently, they are defined using the same inflow conditions, which are the mean wind speed and turbulence field. The overall deficit is computed as in (10), where i varies between 1 and n and represents an upstream turbine.

$$\Delta\overline{U}_{n+1}(t) = max\left(\Delta\overline{U}_i(t)\right)|_{x(n+1)} \tag{10}$$

Larsen Wake Superposition Approach In this final model, each wake is calculated under unsteady inflow conditions, which means that the upstream wakes are considered [12]. The flow field $U(t)$ at the location of the $n^{th}+1$ turbine is estimated like in (11). Every single contribution $U_j(t)$ is calculated at the same location $x(n+1)$. $\overline{U}_j(t)$ represents the inflow condition at the instant t and is calculated according to equation (12).

$$U_{n+1}(t) = \overline{U}_0 + \sum_{j=1}^{n} \left(U_j(t)|_{x(n+1)}^{\tilde{U}_j(t)} - \tilde{U}_j(t)\right) \tag{11}$$

$$\tilde{U}_j(t) = \overline{U}_0 + \sum_{i=1}^{j-1}(U_i(t)|_{x(j)} - \overline{U}_0) \tag{12}$$

3. Physical Model

It was decided to base the physical model on the Jensen model. The choice was made because it is easy to implement in a wider model, usually leading to results that are comparable to the ones of other more complex models [13]. Also, the linear expansion was an extremely important aspect to be able to detect the interceptions between wake and rotor. For the Jensen model, the suggested combination model is the quadratic superposition method [2]. The velocity profile within the wind farm would be then estimated by considering all the wakes created by the turbines and their interaction with the turbines themselves. This way the wind speed at the location of each turbine and therefore the power output could be known. To simplify the aspect of the interaction between wakes and rotors, it was assumed that every time an interception happened, it would be a complete one.

The model as here formulated still needed the definition of two inputs relative to the wind conditions, one describing the wind speed and the other its direction. The input about the wind velocity was chosen according to the value that minimised the accumulated difference between the estimated wind speeds and the ones measured at each turbine. This value (u_{opt}) resulted to be extremely close to the average of the wind speed measured by the turbines (u_{avg}). Consequently, u_{avg} was chosen as the input relative to the wind speed. The characteristic best describing the direction of the wind within the wind farm was thought to be the average wind direction among all the turbines. Therefore, it was chosen to be the input relative to the wind direction.

Before continuing with the estimations of the deficits to which each turbine undergoes, the reliability of the data had to be tested. In order to do so, a new set of power curves was creted using the data themselves and the evolution of the wind field within the wind farm was studied. Several sets of power curves were created but the best one was found to be the one using box plots and medians. In this case, the data were divided into bins according to the wind speeds (each bin was assumed to be of the size of $0.5m/s$) and then the relative box plots were plotted. Sequently, the median of each box was found and a curve passing through them was fitted. By calculating the realtive C_p, it was found that two turbines collected unreliable data relative to the wind velocity. When estimating the power output of these turbines, the power curves provided by the manufacturer were used instead. In order to check the validity of the data relative to the wind direction and the possible presence of significant obstacles, several consecutive quiver plots were created. The observation of these plots allowed to find 4 additional turbines that collected unreliable data. The data collected by the unreliable turbines were not used in any of the calculations.

Finally, all the deficits experienced by the wind turbines were calculated for each speed bin of $0.5m/s$ and angular sector of $22.5°$. These deficits were collected and organised in a $3D$ array, where the three dimensions represented the turbine number, the speed bin and the angular sector. This way, by knowing the two inputs it was possible to find the deficit experienced by each turbine with those wind conditions and consequently estimate the power produced. However, in order for the deficit to be valid the distance between the rotor creating the wake and the one intercepted had to be lower than 40 diameters and the deficit higher than or equal to 0.01.

4. Results of the Physical Model

In order to verify the validity of the results obtained by the simulations made using the physical model several methods were used. The first one was to compare the wind field obtained from the data and the one found with the model. The comparison was made by means of contour plots, which allowed to have an overall view of the wind speed thoughout of the wind farm. Subsequently, the values of the estimated wind speed at each turbine was compared with the measured one in order to have a more precise idea of the discrepancies. These two analysis produced bad results as the outcome of the model did not resemble the measured behaviour. From the latter analysis resulted that according to the model several turbines experience tha same wind speed, which is highly unlikely. In another study, the deficit of the wind turbine experiencing the highest wind speed according to the data was estimated. Obviously, such a deficit should be equal to 0, however in some cases this did not happen. This aspect further highlights the need for improvements relative to this model. Therefore, it was checked if the value of u_{opt} was the correct one. It was confirmed that the minimum found during the minimisation of the accumulated difference was a global one as the function had only one minimum.

Given the lack of results from the physical model, possible reasons were studied. Probably, the most important one is the lack of a study relative to the wind field in the wind farm before the installation of the turbines. If a similar study had been present, possibly no turbines would have experienced the same wind speed according to the model and the outcome would have better resembled the measurements. Another extremely important reason is that when merging the wakes, three phenomena were not considered [14]:

1. if the two wakes are aligned, the resulting one will recover faster than the one produced by a single turbine
2. if they intercept each other only partly, both of them will recover along a more extended distance
3. a certain wake can be forced to move by the pressure field created by another wake

The results lacked consistency also because the data were collected within time gaps of 10 minutes. Such a time gap is very wide for a stochastic resource like wind and therefore does not allow to describe it properly. The other reasons were probably not as important but still played a role in the failure of the physical model. For example, several additional aspects like the variability of the surface roughness length and also its value were not properly considered [15]. Also, not all the effects on the atmosphere were taken into account as the wakes and the rotors have an influence on the value of the stability [16] [17] [18]. It was also found that the Jensen model usually overestimates the value of the wind speed at the centreline [15]. Two lasts aspects that if included could have improved the model are the ground-reflected wakes and the consideration of some obstacles within the wind farm (e.g. met masts).

5. Empirical Model

Given the lack of consistency of the results of the physical model, it was decided to create a new model in order to attempt to better predict the output of the wind farm. Since the aspects that were not considered in the previous model were numerous, it was thought that the best way to include them all was to use a statistical approach based on the data collected by the wind turbines. Two aspects from the previous model were maintained as they were still valid: the power curves and the list of unreliable turbines. The goal of this second model was to create a set of equations giving the power output of each turbine as a result depending on the inputs. In order to have the model properly working, the inputs had to describe the wind conditions (speed and direction) and were consequently chosen to be two. Since this new model attempted to find the wind speed at the location of each turbine based on the undisturbed wind speed, it was thought that the best characteristic describing it was the maximum wind speed measured by any of the reliable turbines. On the other hand, the input relative to the wind direction was chosen to be the average direction of the ones collected by the turbines.

As it has just been said, the goal of this model was to describe the wind speed experienced by any turbine starting from the undisturbed value. In order to do so, the average of the measured wind speeds when certain wind conditions happened was estimated. The wind conditions were defined in bins of the size of $0.5m/s$ and angular sectors of $10°$. This definition was kept for speed values between $3m/s$ and $14m/s$, because beyond the latter value not enough data in order to properly define an average condition were available. From the average speeds it was then possible to estimate the deficits in comparison to the undisturbed (maximum in this case) wind velocity. Similarly to the previous model, these deficits were then gathered in a $3D$ array where the three direction stood for the wind turbine number, the speed bin and the angular sector. This way it was easy to identify the desired deficit when needed during the prediction. In the case of the unrelaible turbines, a weighed average of the deficits estimated for the turbines within a $1km$ radius of the unreliable turbines was done in order to find the deficit. The weights were the reciprocal of the distances from the analysed turbines as they gave more importance to the nearest turbines.

By comparing the deficits obtained using the physical model and the ones produced by the empirical model, it has been noted that the latter values were significantly higher, which is in accordance with the fact of having neglected numerous elements during the formulation of the first model. Like for the physical model, contour plots and comparisons between the wind velocities at the turbines locations were analysed. Once again, the results were not completely accurate but closer than for the physical model. Given the more similar speed values, the prediction of the remaining 3 months was attempted. In this case the results are generally closer to reality compared to the previous analysis, but still there were some cases in which significant discrepancies were present. After simulating the single months, shorter time intervals were studied in order to better note the differences between the predicted and the measured value. An example of the results obtained is reported in Figure 1, where also the mean absolute error throughout the chosen day is reported.

As a next step, a possible reason for the lack of continuity in the consistency of the results was seeked. Therefore, the standard deviation of the average speed previously calculated in order to estimate the deficits was computed. Some of the values found reached very high values, both in absolute and percentage terms. This was therefore thought as the main cause for some of the significant discrepancies noted during the analysis of the

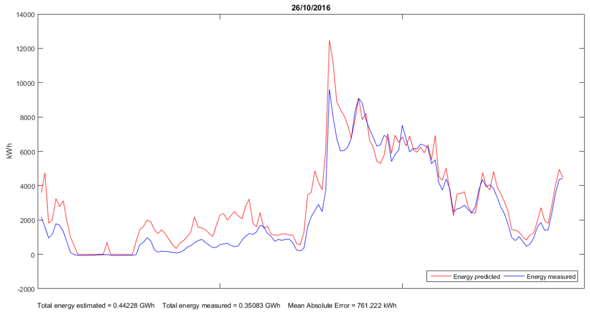

Figure 1: Comparison between measurements and results of the empirical model relative to the power output on $26/10/2016$

3 months. High values for the standard deviation could have been easily predicted as they are in accordance with the stochastic and variable nature of the wind. The results from the first couple of analysis suggested that the empirical model could have been improved. Consequently, it was attempted to include the weather conditions into the model in order to increase the accuracy of the predictions. However, no significant pattern among the several meteorological variable and the wind speed or the power output was found. Therefore, no additional terms were added to the model. Still, some possible improvements were thought for a possible future continuation of the study. First of all, a longer period of time should be used for the training of the model. This way all the possible combinations of wind velocity and direction could verify (which did not happened using only 9 months). Also, using such a limited amount of months might have affected the results because of the seasonal variability and of possible unique conditions happened during the year 2016. An important amelioration would be to include the variability of the wind within the model using statistics. This addition would make the model better at reproducing some of the fluctuations. An alternative in order to improve the prediction of the fluctuations is to use shorter time gaps for the collection of the data as in 10 minutes the wind can vary significantly. Also, if the wind speed changes of a certain amount, the power output will vary of a higher quantity as it is proportional to u^3.

6. Creation of the Series of Equations

The last step relative to the empirical model is the the creation of a series of equations representing the behaviour of the deficits previously found. This operation would allow to quickly know the output of each operating turbine once the conditions of the incoming wind were known. Using the deficits all the power outputs for wind speeds varying between $3m/s$ and $25m/s$ and for every possible angular sector of $10°$ could be estimated for each turbine. Consequently, all the inputs (speeds and directions) and outputs (power) were known. In this case, the best operation in order to find the desired equations is the surface fitting. The first attempts to find the equations were carried out using softwares created exactly for this purpose. Several results with an elevated R^2 value were found using these softwares. However, they were not able to reproduce the oscillations of the data either because the results were almost constant with direction or because the fluctuations had different period and amplitude. Anyway, a couple of interesting functions were found and used to run two optimisations each. The first optimisation aimed at minimising the accumulated difference between the results of the equation and the data, while the goal of the second one was to maximise the value of R^2. The results of the otimisations once again did not resemble the data because of fluctuations.

In a second approach, after finding a function resembling the power curve for each direction (and therefore depending only on the wind speed), a pattern between each single parametre characterising this function and the wind direction had to be found. The function found to best represent the typical behaviour of the power curve is the Five Parameter Logistic Fit, reported in one of his versions in equation (13).

$$f(x) = d + \frac{a - d}{\left(1 + \left(\frac{x}{c}\right)^b\right)^e} \tag{13}$$

where a is the minimum asymptote, b is the steepness of the curve, c indicates the inflection point, d stands for the maximum asymptote and e is the assymmetry factor. If e is equal to 1 the curve will be symmetrical and with 4 free parametres. In an attempt to simplify the fitting and the following step of finding a pattern between the parametres and the direction, three parametres were blocked and only two kept free. Consequently, the parametres a, d and e were fixed at a value of 125 (which was a more or less average value for the power output at $3m/s$), 2300 (which is the rated power of the turbines) and 1, respectively. Therefore, equation (13) became equal to (14).

$$f(x) = 2300 + \frac{125 - 2300}{1 + \left(\frac{x}{c}\right)^b} \tag{14}$$

Afterwards, the equation (14) was fitted to some of the 36 power curves and the relative values of b and c were saved. As a final step the pattern between their values and the direction was seeked. However, no pattern was found and therefore also this second attempt of surface fitting failed.

7. Empirical Power Model

Since in every step of a model usually the margin of error increases, it was thought that a new model with the minimum amount of steps possible could be created. In order to have the least amount of steps possible in the creation of the model, the power output of the whole wind farm was studied and organised depending on the conditions of the incoming wind. In order to keep the error as low as possible, only the time slots when all the turbines were operating were used. This decision however decreased significantly the amount of available data. During the creation of this model all the turbines were considered reliable, as it was not possible to check the validity of the measurements. Since the model was based on the conditions of the undisturbed wind the inputs were chosen to be the same as for the empirical model, maximum wind speed and average direction. The dimensions of the speed bins and the angular sector were maintained respectively at $0.5 m/s$ and $10°$.

In this last model the average output of the whole wind farm when certain wind conditions happened was estimated and collected in a matrix with the angular sectors in the columns and the speed bins in the rows. After having estimated all the average power outputs, it was possible to predict the 3 months left for the validation of the model. Also, the same shorter time intervals analysed in the empirical model were predicted and compared with the previous results. The results of the prediction of the same day examined previously $(26/10/2016)$ are shown in Figure 2.

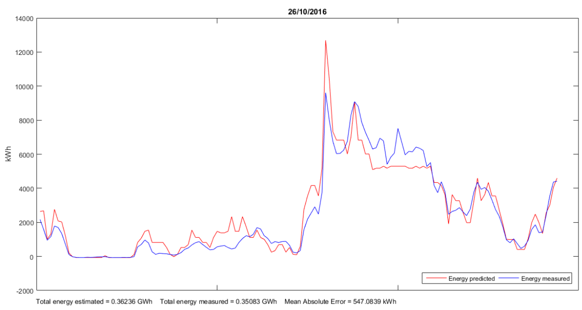

Figure 2: Comparison between measurements and results of the empirical power model relative to the power output on $26/10/2016$

As it was done in the previous case, the standard deviation was studied as it was probably the reason for the discrepancy with the measurements. Again, the standard deviation reached important values in certain cases, highlighting that the variability of the wind is most probably the reason for the discrepancies. In addition to the standard deviation, also the coefficient of variation and the greatest difference between the average power output and any of the measurements were computed. The results highlight even more what has just been stated about the variable nature of the wind. By examining these results, it was noted that the variability of the wind is higher than the possible error introduced by a not operating turbine. For this reason, the results of the empirical power model could be used to predict the output also when not all the turbines are producing power.

As a next step in the analysis, the results of the empirical and of the empirical power model were compared. It was found that the former model tends to predict spikes even though there are none in the measurements. Also, it is better at predicting the power production when it reaches high values and when it fluctuates. On the other hand, the empirical power model is better when the power output is lower and produces overall lower mean absolute errors. Finally, possible ways to improve the model were proposed. The suggestions were basically the same as for the empirical model, as more data should be used in the creation of the model and the variability should be considered either statistically or by using shorter time gaps for the collection of the measurements. In this case the need of more data is even stronger as the training used only 6 months of data because in some cases one or more turbines were not operating.

8. Conclusions

Among the main topics studied, the ones that led to the best outcomes are the study of the empirical and empirical power model. In some cases, they presented good results however, in others the difference between the outcome of the models and the measurements was significant. This aspect highlights the fact that the models as they are are not good enough to precisely predict the output of the wind farm. On the other hand, the aspect producing the worst outcome was the physical model, especially considering the time spent formulating it. By analysing the reasons of its failure, the shortcomings of the Jensen model when working on its own were exposed. Another aspect that did not produced the desired outcome was the creation of the set of equations. For this aspect different methods and approaches were used, but even though the results improved they were never satisfactory. However, it was highlighted that the work conducted so far has a good potential for future improvements, especially considering that in future more data will be available. The work carried out shows also that the best approach for this type of study is the statistical one, as the physical phenomena to be considered are too many.

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